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(Organizador)

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PRÓLOGO

La producción del conocimiento científico y educativo en el campo de las ciencias exactas y naturales se caracteriza, cada vez más, por su capacidad de articular fundamentos teóricos, desarrollos tecnológicos, prácticas formativas y compromisos sociales. En este contexto, el primer volumen de ***Pesquisa e Docência em Ciências Exatas e Naturais*** reúne un conjunto plural de trabajos que evidencian la vitalidad de la investigación contemporánea y la importancia de repensar la docencia como espacio de construcción, mediación y circulación del saber.

Los capítulos que integran esta obra permiten percibir la amplitud de un campo que no se limita a la transmisión de contenidos de disciplinas científicas, sino que se abre a problemas complejos, metodologías diversas y experiencias docentes. La investigación matemática, físico-química y computacional convive aquí con la ingeniería aplicada, la inteligencia artificial, la ética profesional, la educación matemática, la enseñanza de las ciencias, la formación superior y la preservación del conocimiento paleontológico. Esta diversidad temática refleja una visión amplia de las ciencias exactas y naturales, entendidas no solo como áreas de formulación abstracta y experimentación técnica, sino también como prácticas humanas, educativas e institucionales.

El volumen se inicia con trabajos dedicados a la modelización matemática, físico-química y al estudio de sistemas complejos. En este primer conjunto, se abordan problemas relacionados con operadores diferenciales, semigrupos de contracciones, isotermas de adsorción, gases reales, potenciales de Lennard-Jones y Morse, nanoestructuras y configuraciones de mínimo potencial. Estos capítulos destacan la importancia de la modelización, la abstracción y la simulación en la comprensión de fenómenos naturales y materiales.

En un segundo momento, la obra se orienta hacia las tecnologías aplicadas, la ingeniería y los medios digitales en la formación científica. Los trabajos reunidos en esta parte muestran cómo el desarrollo tecnológico puede contribuir tanto a la creación de dispositivos y soluciones aplicadas como a la transformación de los procesos formativos. La presencia de estudios sobre electroestimulación, generación de gráficos vectoriales mediante reconocimiento de voz, aprendizaje profundo e inteligencia artificial en contextos universitarios evidencia la necesidad de repensar la innovación técnica junto con sus implicaciones educativas, epistemológicas y profesionales.

La tercera parte concentra investigaciones orientadas a la docencia, el aprendizaje y la equidad en contextos educativos diversos. Los capítulos analizan cuestiones vinculadas a la ética en ingeniería, a la inclusión en educación matemática, al liderazgo y desempeño docente, a las actitudes hacia la estadística, al aprendizaje basado en proyectos, a la relación entre sueño y aprendizaje, y a la calidad educativa en la formación superior en odontología. En conjunto, estos trabajos subrayan que enseñar ciencias y matemáticas exige mucho más que dominio de la disciplina: requiere sensibilidad pedagógica, reflexión institucional, innovación metodológica y atención a las condiciones reales de aprendizaje de los estudiantes.

Finalmente, el volumen se cierra con una contribución singular dedicada a las ciencias naturales, los acervos fósiles y la preservación del conocimiento paleontológico. A partir de una trayectoria de décadas en la prospección, colección y exhibición de fósiles, este capítulo invita a reflexionar sobre la colaboración entre iniciativas privadas, museos, universidades e instituciones científicas. Su presencia al final de la obra ofrece un cierre significativo, al recordar que la ciencia también depende de la conservación, documentación y accesibilidad de los materiales que permiten reconstruir la historia natural.

De este modo, ***Pesquisa e Docência em Ciências Exatas e Naturais*** propone una lectura que avanza desde los fundamentos científicos y matemáticos hacia las aplicaciones tecnológicas, los medios digitales, los desafíos de la enseñanza y la preservación del patrimonio natural. La obra evidencia que investigar y enseñar están profundamente interrelacionadas: toda investigación produce nuevas preguntas para la formación, y toda práctica docente comprometida puede convertirse en espacio de investigación, innovación y transformación. De hecho, solo se puede enseñar bien a los estudiantes aquel conocimiento que los investigadores antes comprendieron bien.

Esperamos que este volumen contribuya al diálogo entre investigadores, docentes, estudiantes y profesionales interesados en las ciencias exactas y naturales, fortaleciendo una perspectiva integradora, crítica y colaborativa del conocimiento. Que los trabajos aquí reunidos sirvan como punto de partida para nuevas investigaciones, nuevas prácticas pedagógicas y nuevas formas de aproximarse a los desafíos científicos y educativos de nuestro tiempo.

Ramon González Calvet

SUMARIO

DOCENCIA, APRENDIZAJE Y EQUIDAD EN MATEMÁTICAS, CIENCIAS Y FORMACIÓN SUPERIOR

CAPÍTULO 1

CONTRAST WITH THE HILLE-YOSIDA'S THEOREM AND THE CONTRACTION SEMIGROUP FOR AN ODD-ORDER DIFFERENTIAL OPERATOR

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ABSTRACT: This work studies the resolvent properties of an odd-order differential operator defined on periodic Sobolev spaces and compares the results obtained with the Hille-Yosida theorem. We prove that the closed right half-plane is contained in the resolvent set of the operator and that, for complex parameters with positive real part, the norm of the corresponding resolvent operator is bounded by the inverse of that real part. These estimates establish a direct connection with the theory of contraction semigroups. In addition, we analyze the relationship between contraction semigroups and the dissipativeness of their infinitesimal generators. The study also considers particular cases and generalizations for odd-order differential operators, showing how resolvent estimates, dissipativity and semigroup theory are connected in this analytical framework.

KEYWORDS: contraction semigroups; Hille-Yosida theorem; odd-order differential operators; dissipative operators; periodic Sobolev spaces; Fourier analysis.

CONTRASTE CON EL TEOREMA DE HILLE-YOSIDA Y EL SEMIGRUPO DE CONTRACCIONES PARA UN OPERADOR DIFERENCIAL DE ORDEN IMPAR

RESUMEN: En este trabajo se estudian las propiedades resolventes de un operador diferencial de orden impar definido en espacios de Sobolev periódicos y se comparan los resultados obtenidos con el teorema de Hille-Yosida. Se demuestra que el semiplano derecho cerrado está contenido en el conjunto resolvente del operador y que, para parámetros complejos con parte real positiva, la norma del operador resolvente correspondiente está acotada por el inverso de dicha parte real. Estas estimaciones establecen una conexión directa con la teoría de semigrupos de contracciones. Además, se analiza la relación entre los semigrupos de contracciones y la disipatividad de sus generadores infinitesimales. El estudio también considera casos particulares y generalizaciones para operadores diferenciales de orden impar, mostrando cómo las estimaciones resolventes, la disipatividad y la teoría de semigrupos se articulan dentro de este marco analítico.

PALABRAS CLAVE: semigrupos de contracciones; teorema de Hille-Yosida; operadores diferenciales de orden impar; operadores disipativos; espacios de Sobolev periódicos; análisis de Fourier.

MSC 2020: 34K08, 47B44, 47D03,93D23, 46E35, 37L15

1. INTRODUCTION

Recall, H^s_{per} is the periodic Sobolev space with s a real number, $a > 0$ and $\boldsymbol{A} := -\boldsymbol{\partial}_x^3 - \boldsymbol{aI}$ is the infinitesimal generator of the semigroup $\{\boldsymbol{S(t)}\}_{t\geq 0}$ in H^s_{per} that we proved in [2]. We are interested in knowing the resolvent set of A, and bounding the norm of the resolvent operator of A in the sense of obtaining.

$$\|(\boldsymbol{\lambda I} - \boldsymbol{A})^{-1}\| \leq \frac{\mathbf{1}}{\boldsymbol{Re\ \lambda}}, \forall \lambda \in \mathbb{C}_*$$

where $\mathbb{C}_* := \{\boldsymbol{\lambda} \in \mathbb{C}: \boldsymbol{Re\ \lambda} \geq \mathbf{0}\} \subset \boldsymbol{\rho}(\boldsymbol{A})$, which connects us to the Hille-Yosida's Theorem.

In this analysis we obtain much more, we will obtain that the closed right half-plane is contained in the resolvent set of A. And we will generalize the results obtained for an odd-order differential operator.

Furthermore, we will explore the connection between being a contraction semigroup and the dissipativeness of its infinitesimal generator. We will do this study for A and its generalization $\mathcal{A}$. We also obtain results for the case a=0 and their generalizations.

We can cite [2] and [3], where we find some results related to operator A. And we cite [5] and [6] for being a source of inspiration for this work.

The structure of our article is as follows. In section 2, we outline the methodology used and provide the citations for the references consulted. In section 3, we study the resolvent operator of A and $\mathcal{A}$ In section 4, we study the equivalence between -Semigroup of contraction and dissipative operator, and its generalization. Finally, in section 5, we present the conclusions of our study.

2. METHODOLOGY

In this article, we mainly employ [2] and [3] as the theoretical framework. In addition, we use the references [4], [1] and [7] for the Fourier theory in H^s_{per}, and differential calculus in Banach spaces.

Motivated by proving that A is the infinitesimal generator of a contraction semigroup, obtaining important properties of the resolvent of A, we will compare the results obtained with the famous Hille-Yosida's theorem and its Corollary, from [6].

Moreover, we will explore the connection between being a contraction semigroup and the dissipativeness of its infinitesimal generator.

3. RESOLVENT OPERATOR OF A AND $\mathcal{A}$ ON $\mathbb{C}_*$

3.1. RESOLVENT OPERATOR OF A

We introduce the following set

$$\mathbb{C}_* := \{\lambda \in \mathbb{C}, Re\,\lambda > 0\}$$

and we denote $\lambda = Re\,\lambda + i\,Im\,\lambda$.

Thus, we will prove that $\mathbb{C}_* \subset \rho(A)$.

In Theorem 3.1 from [3] we proved that A is dissipative. That is, $Re < Av, v >_s = -a\,\|v\|_s^2 \le 0, \forall v \in D(A)$.

So, we will prove that A dissipative implies (3.1).

Theorem 3.1 *Let* $s \in R$ *and* $a > 0$. *Then the operator* $A := -\partial_x^3 - aI$ *satisfies*

$$\|(\lambda I - A)u\|_s \ge Re\,\lambda\,\|u\|_s, \forall u \in D(A), \lambda \in \mathbb{C}_*. \tag{3.1}$$

Proof.- Indeed, let $u \in D(A)$, $u \neq 0$ and $\lambda \in \mathbb{C}_*$

$$\|(\lambda I - A)u\|_s \|u\|_s \ge |< (\lambda I - A)u, u >_s| \ge Re\{< (\lambda I - A)u, u >_s\}. \tag{3.2}$$

Now,

$$< (\lambda I - A)u, u >_s = \lambda \|u\|_s^2 - < Au, u >_s$$

Then

$$Re < (\lambda I - A)u, u >_s = Re\,\lambda \|u\|_s^2 - Re < Au, u >_s \ge Re\,\lambda\,\|u\|_s^2 \tag{3.3}$$

since $Re < Au, u >_s \le 0$.

Using (3.3) in (3.2) we obtain

$$\|(\lambda I - A)u\|_s \|u\|_s \ge Re\,\lambda\,\|u\|_s^2 .$$

Since $u \neq 0$ implies $\|u\|_s > 0$, then

$$\|(\lambda I - A)u\|_s \ge Re\,\lambda\,\|u\|_s .$$

If $u = 0$, equality in (3.1) holds.

■

Remark 3.1 *From Theorem 3.1 we have that* $\lambda I - A$ *is injective for all* $\lambda \in \mathbb{C}_*$.

Theorem 3.2 *Let* $s \in R$ *and* $a > 0$. *Then the operator* $A := -\partial_x^3 - aI$ *satisfies* $Im(\lambda I - A) = H_{per}^s, \forall \lambda \in \mathbb{C}_*$.

Proof.- Indeed, let $\lambda \in \mathbb{C}_*$, $f \in H^s_{per}$, we will prove that there exists $u \in D(A)$ such that $(\lambda I - A)u = f$.

First, we will obtain the candidate for the solution. To achieve this, we apply the Fourier transform to

$$f = \partial_x^3 u + (\lambda + a)u \tag{3.4}$$

with $u \in D(A)$ and obtain

$$\hat{f}(k) = (ik)^3 \hat{u}(k) + (\lambda + a)\hat{u}(k) = (\lambda + a - ik^3)\hat{u}(k). \tag{3.5}$$

We have

$$\lambda + a - ik^3 \neq 0, \forall k \in \mathbb{Z} \tag{3.6}$$

since $|\lambda + a - ik^3| = \sqrt{(Re\,\lambda + a)^2 + (Im\,\lambda - k^3)^2} > 0, \forall k \in \mathbb{Z}$ where $\lambda = Re\,\lambda + i\,Im\,\lambda$.

From (3.5) and (3.6) we obtain

$$\hat{u}(k) = \frac{\hat{f}(k)}{\lambda + a - ik^3}, \forall k \in \mathbb{Z}. \tag{3.7}$$

From which we get our candidate for the solution of (3.4)

$$u = \left[\left(\frac{\hat{f}(k)}{\lambda + a - ik^3}\right)_{k \in \mathbb{Z}}\right]^{\vee}. \tag{3.8}$$

Second, we will prove

$$u \in H^{s+3}_{per} \text{ and } \exists \mathcal{M} > 0,\ \|u\|_s \leq \|u\|_{s+3} \leq \sqrt{\mathcal{M}}\,\|f\|_s. \tag{3.9}$$

Indeed, from (3.8) we have

$$\begin{aligned}\|u\|^2_{H^{s+3}_{per}} &= 2\pi \sum_{k=-\infty}^{+\infty} (1 + k^2)^{s+3}.|\hat{u}(k)|^2 \\ &= 2\pi \sum_{k=-\infty}^{+\infty} (1 + k^2)^{s+3} \left|\frac{\hat{f}(k)}{\lambda + a - ik^3}\right|^2 \\ &= 2\pi \sum_{k=-\infty}^{+\infty} (1 + k^2)^{s} |\hat{f}(k)|^2 \underbrace{\frac{(1 + k^2)^3}{(Re\,\lambda + a)^2 + (Im\,\lambda - k^3)^2}}_{\mathfrak{S}(k):=}\end{aligned} \tag{3.10}$$

where $\lambda = Re\,\lambda + i\,Im\,\lambda$.

We have that there exists $M > 0$ such that

$$0 < \mathfrak{S}(k) \leq \mathcal{M}, \forall k \in \mathbb{Z}. \quad (3.11)$$

Using (3.11) in (3.10) we get

$$\|u\|_{s+3}^2 \leq \mathcal{M}.\, 2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s \left|\hat{f}(k)\right|^2 = \mathcal{M}\|f\|_s^2.$$

As $H_{per}^{s+3} \subset H_{per}^s$ and with continuous immersion we get (3.9).

Finally, we will prove $(\lambda I - A)U = f$ in H_{per}^s . Indeed,

$$\|(\lambda I - A)u - f\|_s^2 = 2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s . \left| (\lambda + a - ik^3)\hat{u}(k) - \hat{f}(k)\right|^2$$

$$= 2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s . \left| (\lambda + a - ik^3) \cdot \frac{\hat{f}(k)}{\lambda + a - ik^3} - \hat{f}(k)\right|^2$$

$$= 2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s \left|\hat{f}(k) - \hat{f}(k)\right|^2 = 0 . \quad (3.12)$$

That is, $(\lambda I - A)U - f = 0$.

■

Theorem 3.3 *Let* $s \in R$ *and* $a > 0$. *Then* $(\lambda I - A)^{-1}: H_{per}^s \to D(A)$ *is a bounded linear operator for each* $\lambda \in \mathbb{C}_*$ *and it satisfies:* $\exists M > 0$ *such that*

$$\left\|(\lambda I - A)^{-1}\right\| \leq \sqrt{\mathcal{M}}\, , \lambda \in \mathbb{C}_*.$$

Proof.- The inequality (3.1) implies $\lambda I - A$ injective for all $\lambda \in \mathbb{C}_*$. Thus, from Theorem 3.2, we have that $\lambda I - A$ is bijective then there exists $(\lambda I - A)^{-1}: H_{per}^s \to D(A)$ which is linear with $(\lambda I - A)^{-1} f = u$ and from (3.9) it satisfies

$$\left\|(\lambda I - A)^{-1} f\right\|_s \leq \left\|(\lambda I - A)^{-1} f\right\|_{s+3} \leq \sqrt{\mathcal{M}} \|f\|_s \, , \forall f \in H_{per}^s \, , \forall\, \lambda \in \mathbb{C}_* . \quad \mathbf{(3.13)}$$

Then $(\lambda I - A)^{-1}$ is a bouded operator such that $\left\|(\lambda I - A)^{-1}\right\| \leq \sqrt{\mathcal{M}}$ holds for all $\lambda \in \mathbb{C}_*$.

■

Remark 3.2 *Also from Theorems 3.1 and 3.2 we obtain* $\left\|(\lambda I - A)^{-1} f\right\|_s \leq \frac{1}{Re\, \lambda} \|f\|_s \, , \forall f \in H_{per}^s \, , \forall\, \lambda \in \mathbb{C}_*$ *. That is,* $\left\|(\lambda I - A)^{-1}\right\| \leq \frac{1}{Re\, \lambda}, \forall\, \lambda \in \mathbb{C}_*$.

Therefore, we obtain the following result.

Corollary 3.1 *Based on the hypothesis of the previous Theorem, we get* $\mathbb{C}_* \subset \rho(A)$ *and*

$$\left\|(\lambda I - A)^{-1}\right\| \leq \frac{1}{Re\, \lambda}\, , \forall\, \lambda \in \mathbb{C}_* .$$

Proof.- From Remark 3.2 or Theorems 3.2 and 3.3 we obtain the result.

■

Corollary 3.2 *Since* $\mathbb{R}^{+} \subset \mathbb{C}_{*}$ *then* $\mathbb{R}^{+} \subset \rho(A)$ *and* $\left\|(\lambda I - A)^{-1}\right\| \leq \frac{1}{\lambda}, \forall \lambda > 0$.

Proof.- From Corollary 3.1 we obtain the result.

■

Remark 3.3 *Based on the hypothesis of the previous Theorem, using Corollary 3.1 with Theorem 3.10 or Corollary 3.4 from [3], we have* $i\mathbb{R} \cup \mathbb{C}_{*} = \{\lambda \in \mathbb{C}: Re\,\lambda \geq 0\} \subset \rho(A)$ *and*

$$\left\|(\lambda I - A)^{-1}\right\| \leq \begin{cases} \dfrac{1}{Re\,\lambda}, & \lambda \in \mathbb{C}_{*} \\ \dfrac{1}{a}, & \lambda \in i\mathbb{R}. \end{cases}$$

That is, the closed right half-plane is contained in the resolvent set of A.

3.2. RESOLVENT OPERATOR OF $\mathcal{A}$

In Theorem 3.13 from [3] we proved that $\mathcal{A}$ is dissipative. That is, $Re < \mathcal{A}v, v >_s = -a\,\|v\|_s^2 \leq 0, \forall v \in D(\mathcal{A})$.

So, we will prove that $\mathcal{A}$ dissipative implies (3.14).

Theorem 3.4 *Let* $s \in R$, $a > 0$ *and n is an odd number such that* $n-1$ *is not multiple of four. Then the operator* $\mathcal{A} := -\partial_x^n - aI$ *satisfies*

$$\|(\lambda I - \mathcal{A})u\|_s \geq Re\,\lambda \|u\|_s, \forall u \in D(\mathcal{A}), \forall \lambda \in \mathbb{C}_{*}. \qquad (3.14)$$

Proof.- Its proof is analogous to the demonstration of Theorem 3.1.

■

Remark 3.4 *From Theorem 3.4 we have that* $\lambda I - \mathcal{A}$ *is injective for all* $\lambda \in \mathbb{C}_{*}$.

Theorem 3.5 *Let* $s \in R$, $a > 0$ *and n is an odd number such that* $n-1$ *is not multiple of four. Then the operator* $\mathcal{A} := -\partial_x^n - aI$ *satisfies* $Im(\lambda I - \mathcal{A}) = H_{per}^s, \forall \lambda \in \mathbb{C}_{*}$.

Proof.- Its proof is similar to the demonstration of Theorem 3.2.

■

Theorem 3.6 *Let* $s \in R$, $a > 0$ *and n is an odd number such that* $n-1$ *is not multiple of four. Then* $(\lambda I - \mathcal{A})^{-1}: H_{per}^s \to D(\mathcal{A})$ *is a bounded linear operator for each* $\lambda \in \mathbb{C}_{*}$ *and it satisfies:* $\exists \mathbb{M} > 0$,

$$\left\|(\lambda I - \mathcal{A})^{-1}\right\| \leq \sqrt{\mathbb{M}}, \lambda \in \mathbb{C}_{*}.$$

Proof.- Its proof is analogous to the demonstration of Theorem 3.3.

■

Remark 3.5 *Also from Theorems 3.4 and 3.5 we obtain* $\|(\lambda I-\mathcal{A})^{-1}f\|_s \leq \frac{1}{Re\,\lambda}\|f\|_s, \forall f\in H^s_{per}, \forall\lambda\in\mathbb{C}_*$. *That is,* $\|(\lambda I-\mathcal{A})^{-1}\|\leq\frac{1}{Re\,\lambda}, \forall\,\lambda\in\mathbb{C}_*$.

Therefore, we obtain the following result.

Corollary 3.3 *Based on the hypothesis of the previous Theorem, we obtain* $\mathbb{C}_*\subset\rho(\mathcal{A})$ *and*

$$\|(\lambda I-\mathcal{A})^{-1}\|\leq\frac{1}{Re\,\lambda}, \forall\lambda\in\mathbb{C}_*.$$

Proof.- From Remark 3.5 or Theorems 3.5 and 3.6 we obtain the result.

■

Corollary 3.4 *Since* $\mathbb{R}^+\subset\mathbb{C}_*$ *then* $\mathbb{R}^+\subset\rho(A)$ *and* $\|(\lambda I-\mathcal{A})^{-1}\|\leq\frac{1}{\lambda}, \forall\,\lambda>0$.

Proof.- From Corollary 3.3 we obtain the result.

■

Remark 3.6 *Based on the hypothesis of the previous Theorem, using Corollary 3.3 with Theorem 3.22 or Corollary 3.8 from [3], we have* $i\mathbb{R}\cup\mathbb{C}_*=\{\lambda\in\mathbb{C}: Re\,\lambda\geq 0\}\subset\rho(\mathcal{A})$ *and*

$$\|(\lambda I-\mathcal{A})^{-1}\|\leq\begin{cases}\frac{1}{Re\,\lambda}, & \lambda\in\mathbb{C}_*\\ \frac{1}{a}, & \lambda\in i\mathbb{R}.\end{cases}$$

That is, the closed right half-plane is contained in the resolvent set of $\mathcal{A}$.

Remark 3.7 Similar *results are also obtained when n is an odd number such that $n-1$ is multiple of four, in this case, note that* $(ik)^n=ik^n, \forall k\in\mathbb{Z}$.

3.3. CASE a=0

Theorem 3.7 *Let $s\in R$ and a=0 Then the operator* $A:=-\partial_x^3-aI=-\partial_x^3$ *is dissipative on* H^s_{per} *where* $D(A)=H^{s+3}_{per}$. *That is,*

$$Re\{<Au,u>_s\}=0, \forall u\in H^{s+3}_{per}. \tag{3.15}$$

Moreover, $<Au,u>_s=i\delta$, $\forall u\in H^{s+3}_{per}$ where

$$\mathbb{R}\ni\delta:=2\pi\sum_{k=-\infty}^{+\infty}(1+k^2)^s k^3|\hat{u}(k)|^2$$

Proof.- Let $u\in H^{s+3}_{per}$,

$$< \boldsymbol{Au}, \boldsymbol{u} >_s \;=\; < -\partial_x^3 u, u >_s = \underbrace{i.2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s k^3 |\hat{u}(k)|^2}_{\delta:=} . \tag{3.16}$$

At this point, we will prove the convergence of the series (3.16). Indeed, using the inequality: $|k|^3 \le |k|^6 = (|k|^2)^3 \le (1+|k|^2)^3$ and $u \in H_{per}^{s+3}$, we have

$$\left|\sum_{k=-\infty}^{+\infty} (1+k^2)^s k^3 |\hat{u}(k)|^2\right| \le \sum_{k=-\infty}^{+\infty} (1+k^2)^s |k|^3 |\hat{u}(k)|^2$$

$$\le \sum_{k=-\infty}^{+\infty} (1+k^2)^s (1+|k|^2)^3 |\hat{u}(k)|^2$$

$$= \sum_{k=-\infty}^{+\infty} (1+k^2)^{s+3} |\hat{u}(k)|^2 = \frac{1}{2\pi} \|u\|_{s+3}^2 < \infty .$$

Then the series (3.16) is convergent, that is,

$$< Au, u >_s = i\delta \text{ , with } \delta \in R . \tag{3.17}$$

Finally, from equality (3.17) we obtain $\mathbf{Re}\{< \mathbf{Au}, \mathbf{u} >_s\} = \mathbf{0}$, for all $u \in H_{per}^{s+3}$

■

Theorem 3.8 *Let* $\boldsymbol{s} \in \boldsymbol{R}$*, then the operator* $\boldsymbol{A} := -\boldsymbol{\partial}_x^3$ *satisfies inequality (3.1).*

Proof.- *The proof is similar to the demonstration of Theorem 3.1 where the Theorem 3.7 is used.*

■

Remark 3.8 *From Theorem 3.8 we have that* $\boldsymbol{\lambda I} - \boldsymbol{A}$ *is injective for all* $\boldsymbol{\lambda} \in \mathbb{C}_*$

Theorem 3.9 *Let* $\boldsymbol{s} \in \boldsymbol{R}$*, then the operator* $\boldsymbol{A} := -\boldsymbol{\partial}_x^3$ *satisfies* $\boldsymbol{Im}(\boldsymbol{\lambda I} - \boldsymbol{A}) = \boldsymbol{H}_{per}^s$, $\forall\, \boldsymbol{\lambda} \in \mathbb{C}_*$

Proof.- *The proof is similar to the demonstration of Theorem 3.2.*

■

Theorem 3.10 *Let* $\boldsymbol{s} \in \boldsymbol{R}$ *and* $\boldsymbol{a=0}$ *Then* $(\boldsymbol{\lambda I} - \boldsymbol{A})^{-1}: \boldsymbol{H}_{per}^s \to \boldsymbol{D}(\boldsymbol{A})$ *is a bounded linear operator for each* $\boldsymbol{\lambda} \in \mathbb{C}_*$ *and it satisfies:* $\exists \mathfrak{B} > \mathbf{0}$ *such that*

$$\|(\boldsymbol{\lambda I} - \boldsymbol{A})^{-1}\| \le \sqrt{\mathfrak{B}}, \boldsymbol{\lambda} \in \mathbb{C}_* .$$

Proof.- The proof is consequence of Theorems 3.8 and 3.9.

■

Remark 3.9 *Also from Theorems 3.8 and 3.9 we obtain* $\|(\boldsymbol{\lambda I} - \boldsymbol{A})^{-1} \boldsymbol{f}\|_s \le \frac{\mathbf{1}}{\boldsymbol{Re\,\lambda}} \|\boldsymbol{f}\|_s$, $\forall \boldsymbol{f} \in \boldsymbol{H}_{per}^s$, $\forall\, \boldsymbol{\lambda} \in \mathbb{C}_*$. *That is,*

$$\left\|(\lambda I - A)^{-1}\right\| \leq \frac{1}{Re\,\lambda}, \forall\, \lambda \in \mathbb{C}_* .$$

Therefore, we obtain the following result.

Corollary 3.5 *Based on the hypothesis of the previous Theorem, we get* $\mathbb{C}_* \subset \rho(A)$ *and*

$$\left\|(\lambda I - A)^{-1}\right\| \leq \frac{1}{Re\,\lambda}, \forall\, \lambda \in \mathbb{C}_* .$$

That is, the open right half-plane is contained in the resolvent set of A.

Proof.- From Remark 3.9 or Theorems 3.9 and 3.10 we obtain the result.

■

Corollary 3.6 *Since* $\mathbb{R}^+ \subset \mathbb{C}_*$ *then* $\mathbb{R}^+ \subset \rho(A)$ *and* $\left\|(\lambda I - A)^{-1}\right\| \leq \frac{1}{\lambda}, \forall\, \lambda > 0$.

Proof.- From Corollary 3.5 we obtain the result.

■

Remark 3.10 *If* $a=0$ *then* $0 \notin \rho\,(A)$

Remark 3.11 *If* $a=0$ *then the* $\{S(t)\}_{t\geq 0}$ *semigroup is not exponentially stable (See [1]).*

Next, we will generalize the results obtained.

Theorem 3.11 *Let* $s \in R$ *and* $a=0$ *and* n *is an odd number such that* n *-1 is not multiple of four. Then the operator* $\mathcal{A} := -\partial_x^n - aI = -\partial_x^n$ *is dissipative on* H^s_{per} *where* $D(\mathcal{A}) = H^{s+n}_{per}$. *That is,*

$$Re\{< \mathcal{A}u, u >_s\} = 0, \forall u \in H^{s+n}_{per}. \tag{3.18}$$

Moreover, $< \mathcal{A}u, u >_s = i\delta,\ \forall u \in H^{s+n}_{per}$ where

$$\mathbb{R} \ni \delta := 2\pi \sum_{k=-\infty}^{+\infty} \left(1+k^2\right)^s k^n |\hat{u}(k)|^2$$

Proof.- The proof is similar to the demonstration of Theorem 3.8.

■

Theorem 3.12 *Let* $s \in R$ *and n is an odd number such that n-1 is not multiple of four. Then the operator* $\mathcal{A} := -\partial_x^n$ *satisfies inequality (3.14).*

Proof.- *The proof is similar to the demonstration of Theorem 3.1 where the Theorem 3.11 is used.*

■

Remark 3.12 *From Theorem 3.12 we have that* λI-$\mathcal{A}$ *is injective for all* $\lambda \in \mathbb{C}_*$.

Theorem 3.13 *Let* $s \in R$ *and n is an odd number such that n-1 is not multiple of four. Then the operator* $\mathcal{A} := -\partial_x^n$ *satisfies* $Im(\lambda I - \mathcal{A}) = H_{per}^s$, $\forall \lambda \in \mathbb{C}_*$.

Proof.- *The proof is similar to the demonstration of Theorem 3.2.*

■

Theorem 3.14 *Let* $s \in R$, $a=0$ *and n is an odd number such that n-1 is not multiple of four. Then* $(\lambda I - \mathcal{A})^{-1}: H_{per}^s \to D(\mathcal{A})$ *is a bounded linear operator for each* $\lambda \in \mathbb{C}_*$ *and it satisfies:* $\exists \mathcal{L} > 0$ *such that*

$$\|(\lambda I - \mathcal{A})^{-1}\| \leq \sqrt{\mathfrak{L}}, \lambda \in \mathbb{C}_* .$$

Proof.- The proof is consequence of Theorems 3.12 and 3.13.

■

Remark 3.13 *Also from Theorems 3.12 and 3.13 we obtain* $\|(\lambda I - \mathcal{A})^{-1} f\|_s \leq \frac{1}{Re\,\lambda} \|f\|_s$, $\forall f \in H_{per}^s$, $\forall \lambda \in \mathbb{C}_*$. *That is,*

$$\|(\lambda I - \mathcal{A})^{-1}\| \leq \frac{1}{Re\,\lambda}, \forall \lambda \in \mathbb{C}_* .$$

Therefore, we obtain the following result.

Corollary 3.9 *Based on the hypothesis of the previous Theorem, we get* $\mathbb{C}_* \subset \rho(\mathcal{A})$ *and*

$$\|(\lambda I - \mathcal{A})^{-1}\| \leq \frac{1}{Re\,\lambda}, \forall \lambda \in \mathbb{C}_* .$$

That is, the open right half-plane is contained in the resolvent set of $\mathcal{A}$ *.*

Proof.- From Remark 3.13 or Theorems 3.13 and 3.14 we obtain the result.

■

Corollary 3.10 *Since* $\mathbb{R}^+ \subset \mathbb{C}_*$ *then* $\mathbb{R}^+ \subset \rho(\mathcal{A})$ *and* $\|(\lambda I - \mathcal{A})^{-1}\| \leq \frac{1}{\lambda}, \forall \lambda > 0$ *.*

Proof.- From Corollary 3.9 we obtain the result.

■

Remark 3.14 *If* $a=0$ *then* $0 \notin \rho(\mathcal{A})$.

Remark 3.15 *If* $a=0$ *then the* $\{\mathcal{T}(t)\}_{t\geq 0}$ *semigroup is not exponentially stable (See [1]).*

Remark 3.16 Analogous *results are also obtained when* n *is an odd number such that* n *-1 is multiple of four, in this case, note that* $(ik)^n = ik^n, \forall k \in \mathbb{Z}$.

3.4. COMMENTS

We have already the following statements, where $A = -\partial_x^3 - aI$ is a linear operator. Remember that items 1 and 2 were proved in [2] and item 3 in [3] for $a>0$. On the other hand, items 1 and 2 were proved in [1] for $a=0$.

1. A is closed and $\overline{D(A)} = H_{per}^s$, where $D(A)= H_{per}^{s+3}$.
2. A is the infinitesimal generator of a C_o semigroup of contractions $\{S(t)\}_{t\geq 0}$.
3. $\mathbb{R}^+ \subseteq \rho(A)$ and for every $\lambda>0$

$$\|(\lambda I - A)^{-1}\| \leq \frac{1}{\lambda}$$

4. $\mathbb{C}_* \subset \rho\,(A)$ *and for every* $\lambda \in \mathbb{C}_*$

$$\|(\lambda I - A)^{-1}\| \leq \frac{1}{Re\,\lambda}$$

Now, we will analyze its connection with Hille-Yosida's Theorem 3.1 from [6]. By Hille-Yosida's Theorem we have that item 2 implies items 1 and 3 and that the reciprocal is fulfilled. We proved it without resorting to this Theorem. Note also that using item 4 we get item 3.

Another commonly used result is the Corollary 3.6 from [6], which tells us that item 2 implies item 4. Note that we naturally proved it without resorting to this Corollary.

This analysis also applies to $\mathcal{A} = -\partial_x^n - aI$ when n is odd. Thus, we have compared our results with two important results: Hille-Yosida's Theorem and its Corollary, that motivated this study.

This study is limited to operators of type: $\mathcal{A} = -\partial_x^n - aI$ in H_{per}^s where n is an odd number; analogous to this can be established for n multiple of four in which, for example, an equality is obtained: $Re < \mathcal{A}v, v >_s = -a\,\|v\|_s^2 - \left\|\partial_x^{n/2} v\right\|_s^2 \leq 0, \forall v \in D(\mathcal{A})$, unlike the odd case in which the equality $Re < \mathcal{A}v, v >_s = -a\,\|v\|_s^2 \leq 0, \forall v \in D(\mathcal{A})$ is obtained.

Let us remember that in the case studied we do obtain the explicit form of the solution, which is a great advantage; whereas Hille-Yosida's Theorem allow us to know theoretically that exists a solution, without knowing the explicit form of the solution (as in the applications in Liu-Zheng [5] or Pazy [6]). Hence the relevance of this important Theorem.

Finally, to make the reading self-sufficient and quick, we will state a important result that we frequently use to determine whether an operator is the infinitesimal generator of a contraction semigroup and an important Corollary that expands the resolvent set and provides a bound for the resolvent operator.

Theorem 3.15 (Hille-Yosida) *A (unbounded)linear operator* B *is the infinitesimal generator of a* C_o *semigroup of contractions* $\{T(t)\}_{t\geq 0}$ *if and only if*

(i) B *is closed and* $\overline{D(B)} = X$.

(ii) the resolvent set $\rho(B)$ *of* B *contains* $\mathbb{R}^+$ *and for every* $\lambda>0$

$$\|(\lambda I - B)^{-1}\| \leq \frac{1}{\lambda}$$

Proof.- We cite Theorem 3.1 from [6].

■

Corollary 3.11 *Let B be the infinitesimal generator of a* C_o *semigroup of contractions* $\{T(t)\}_{t\geq 0}$. *The resolvent set of A contains the open right half-plane, i.e.* $\rho(B) \supseteq \{\lambda \in \mathbb{C}: Re\lambda>0\}$ *and for such* λ.

$$\|(\lambda I - B)^{-1}\| \leq \frac{1}{Re\ \lambda}$$

Proof.- We cite Corollary 3.6 from [6].

■

4. EQUIVALENCE BETWEEN C_o OF CONTRACTION AND DISSIPATIVE OPERATOR

Theorem 4.1 *Let* H^s_{per} *be a Hilbert space and the* C_o *- semigroup* $\{S(t)\}_{t\geq 0}$ *from [2] such that* A *is its infinitesimal generator, then the following equivalence holds:* $\{S(t)\}_{t\geq 0}$ *is of contraction if and only if* A *is dissipative, that is,* $Re < Av, v >_s = -a\|v\|_s^2 \leq 0, \forall v \in H^{s+3}_{per} = D(A)$

Proof.- As $\{S(t)\}_{t\geq 0}$ is of contraction then

$$\|S(t)\| \leq 1, \forall t \geq 0. \tag{4.1}$$

Then, let $u \in H^s_{per} - \{0\}$,

$$\langle S(h)u - u, u\rangle_s = \langle S(h)u, u\rangle_s - \langle u, u\rangle_s = \langle S(h)u, u\rangle_s - \|u\|_s^2 \tag{4.2}$$

Taking part real, we obtain

$$Re\ \langle S(h)u - u, u\rangle_s = Re\ \langle S(h)u, u\rangle_s - \|u\|_s^2, \tag{4.3}$$

Using $Re(z) \leq |Re(z)| \leq |z|, \forall z \in \mathbb{C}$, Hölder's inequality and (4.1), we obtain

$$Re\ \langle S(h)u - u, u\rangle_s \leq |\langle S(h)u, u\rangle_s| - \|u\|_s^2 \leq \|S(h)u\|_s\|u\|_s - \|u\|_s^2 \leq \|u\|_s^2 - \|u\|_s^2 = 0 \tag{4.4}$$

Therefore,

$$Re\ \langle S(h)u-u,u\rangle_s \le 0, \forall u \in H^s_{per}. \tag{4.5}$$

In particular,

$$Re\ \langle S(h)v-v,v\rangle_s \le 0, \forall v \in H^{s+3}_{per} = D(A). \tag{4.6}$$

If $v \in D(A)$ then

$$Re\ \langle \frac{S(h)v-v}{h}, v\rangle_s \le 0, \forall h>0. \tag{4.7}$$

As $\frac{S(h)v-v}{h} \to Av$ when $h \to 0^+$ and $\langle \cdot,\cdot\rangle_s$ is continuous then

$$\langle \frac{S(h)v-v}{h}, v\rangle_s \to \langle Av,v\rangle_s \quad \text{when } h\to 0^+. \tag{4.8}$$

Therefore,

$$\text{Re}\ \langle \frac{S(h)v-v}{h}, v\rangle_s \to Re\ \langle Av,v\rangle_s \quad \text{when } h\to 0^+. \tag{4.9}$$

$$\text{Im}\ \langle \frac{S(h)v-v}{h}, v\rangle_s \to Im\ \langle Av,v\rangle_s \quad \text{when } h\to 0^+. \tag{4.10}$$

Using (4.9) in (4.7), we get

$$Re\ \langle Av,v\rangle_s \le 0.$$

Then

$$Re\ \langle Av,v\rangle_s \le 0, \forall v \in D(A). \tag{4.11}$$

Now, we want to know $Re\ \langle Av,v\rangle_s$ explicitly; **what is** $Re\ \langle Av,v\rangle_s$ **?**

Let $h>0$, we know

$$\langle \frac{S(h)u-u}{h}, u\rangle_s = \frac{2\pi}{h}\sum_{k=-\infty}^{+\infty}(1+k^2)^s\{e^{(ik^3-a)h}\,\hat{u}(k)-\hat{u}(k)\}\overline{\hat{u}(k)}$$

$$= 2\pi \sum_{k=-\infty}^{+\infty}(1+k^2)^s\frac{\{e^{(ik^3-a)h}-1\}}{h}|\hat{u}(k)|^2$$

for $u \in H^s_{per}$.

In particular,

$$\langle \frac{S(h)v-v}{h}, v\rangle_s = 2\pi \sum_{k=-\infty}^{+\infty}(1+k^2)^s\frac{\{e^{(ik^3-a)h}-1\}}{h}|\hat{v}(k)|^2 \tag{4.12}$$

for $v \in H^{s+3}_{per} = D(A)$.

Using the Weierstrass M-Test we have that the series (4.12) converges uniformly and consequently it is possible to exchange limits and obtain

$$\lim_{h\to 0^+} \langle \frac{S(h)v - v}{h}, v\rangle_s = 2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s \lim_{h\to 0^+} \left\{\frac{e^{(ik^3-a)h} - 1}{h}\right\} |\widehat{v}(k)|^2$$

$$= 2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s \{ik^3 - a\} \, |\widehat{v}(k)|^2$$

$$= i\, 2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s k^3 \, |\widehat{v}(k)|^2 - a\|v\|_s^2 \qquad (4.13)$$

As $\langle\cdot,\cdot\rangle_s$ is continuous and $v \in D(A)$, from (4.13) we obtain

$$\langle Av, v\rangle_s = i\, 2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s k^3 \, |\widehat{v}(k)|^2 - a\|v\|_s^2 \qquad (4.14)$$

Taking the real part of (4.14), we get

$$Re\,\langle Av, v\rangle_s = -\,a\|v\|_s^2 \qquad (4.15)$$

and therefore $Re\,\langle Av,v\rangle_s \leq 0$.

Thus, (4.15) is the answer to our question. Furthermore, it is observed that Theorem 3.1 from [3] also answers our question.

Reciprocally, as A is dissipative then

$$Re\,\langle Av, v\rangle_s \leq 0, \forall v \in H_{per}^{s+3} = D(A). \qquad (4.16)$$

Let $\vartheta \in D\,(A)$, we define $u(t) := S(t)\vartheta$, then u satisfies:

$$\left| \begin{array}{l} u_t = Au \\ u(0) = \vartheta \end{array} \right.$$

And using (4.16)

$$\frac{1}{2}\frac{\partial}{\partial t}\{\|u\|_s^2\} = \frac{1}{2}\frac{\partial}{\partial t}\{\langle u, u\rangle_s\} = Re\,\langle u_t, u\rangle_s = Re\,\langle Au, u\rangle_s \leq 0$$

since $u(t) \in D(A)$. That is, $\frac{\partial}{\partial t}\{\|u(t)\|_s^2\} \leq 0$. Thus, $\|u(\cdot)\|_s^2$ is non-increasing. Then $\|u(\cdot)\|_s$ is non-increasing.

So,

$$\|S(t)\vartheta\|_s = \|u(t)\|_s \leq \|u(0)\|_s = \|\vartheta\|_s \,, \forall t \geq 0.$$

That is,

$$\|S(t)\vartheta\|_s \leq \|\vartheta\|_s, \forall \vartheta \in D(A), \forall t \geq 0. \quad (4.17)$$

By density, we obtain

$$\|S(t)\varphi\|_s \leq \|\varphi\|_s, \forall \varphi \in H^s_{per}, \forall t \geq 0. \quad (4.18)$$

Finally, from (4.18) we get $\|S(t)\| \leq 1, \forall t \geq 0$.

■

Corollary 4.1 *If* $\{S(t)\}_{t\geq 0}$ *is of contraction then* $Re\,\langle Av, v\rangle_s = -a\|v\|_s^2 \leq 0$, $\forall v \in H^{s+3}_{per} = D(A)$ *Moreover,* $\langle Av, v\rangle_s = i\delta - a\|v\|_s^2$, $\forall v \in H^{s+3}_{per}$ *where*

$$\mathbb{R} \ni \delta = 2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s k^3 \, |\hat{v}(k)|^2$$

Theorem 4.2 *Let* H^s_{per} *be a Hilbert space,* $a=0$ *and the* C_o*- semigroup* $\{S(t)\}_{t\geq 0}$ *from [1] such that A is its infinitesimal generator, then the following equivalence holds:* $\|S(t)\|=1, \forall t \geq 0$ *if and only if* $Re < Av, v >_s = 0, \forall v \in H^{s+3}_{per} = D(A)$.

Proof.- The proof is similar to demonstrate of Theorem 4.1

■

Next, we will generalize the results obtained.

Theorem 4.3 *Let* H^s_{per} *be a Hilbert space, n is an odd number such that* $n-1$ *is not multiple of four and the* C_o *- semigroup* $\{T(t)\}_{t\geq 0}$ *from [2] such that* $\mathcal{A}$ *is its infinitesimal generator, then the following equivalence holds:* $\{T(t)\}_{t\geq 0}$ *is of contraction if and only if* $\mathcal{A}$ *is dissipative, that is,* $Re < \mathcal{A}v, v >_s = -a\|v\|_s^2 \leq 0, \forall v \in H^{s+n}_{per} = D(\mathcal{A})$.

Proof.- The proof is analogous to the demonstration of Theorem 4.1.

■

Corollary 4.2 *If* $\{T(t)\}_{t\geq 0}$ *is of contraction then* $Re\,\langle \mathcal{A}v, v\rangle_s = -a\|v\|_s^2 \leq 0$, $\forall v \in H^{s+n}_{per} = D(\mathcal{A})$.

Moreover, $\langle \mathcal{A}v, v\rangle_s = i\delta - a\|v\|_s^2$, $\forall v \in H^{s+n}_{per}$ *where*

$$\mathbb{R} \ni \delta = 2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s k^n \, |\hat{v}(k)|^2.$$

Theorem 4.4 *Let* H^s_{per} *be a Hilbert space,* $a=0$ *n is an odd number such that* $n-1$ *is not multiple of four and the* C_o*- semigroup* $\{T(t)\}_{t\geq 0}$ *from [1] such that* $\mathcal{A}$ *is its* *infinitesimal generator, then the following equivalence holds:* $\|T(t)\|=1, \forall t\geq 0$ *if and only if* $Re < \mathcal{A}v, v >_s = 0, \forall v \in H_{per}^{s+n} = D(\mathcal{A})$.

Proof.- The proof is similar to demonstrate of Theorem 4.1

■

Remark 4.1 *Similar results to Theorem 4.3, Corollary 4.2 and Theorem 4.4 are obtained when n is an odd number such that* $n-1$ *is multiple of four, where*

$$\mathbb{R} \ni \delta = -2\pi \sum_{k=-\infty}^{+\infty} (1+k^2)^s k^n |\hat{v}(k)|^2.$$

5. CONCLUSIONS

Using properties of periodic Sobolev spaces, we proved that the closed right half-plane is contained in the resolvent set of odd-order differential operator A with $a>0$, and that the norm of the resolvent operator of A on λ with positive real part is bounded by the inverse of the real part of λ, which connects us to the Hille-Yosida's Theorem. Also, we proved that the open right half-plane is contained in the resolvent set of odd-order differential operator A when a=0. Likewise, it was observed that if one dissipation is eliminated (a = 0), then the semigroup is not exponentially stable. Thus, we have compared our results with two important results: Hille-Yosida's Theorem and its Corollary, that motivated this study. Furthermore, we explored the connection between being a contraction semigroup and the dissipativeness of its infinitesimal generator for $a\geq 0$. Finally, we generalized the results obtained.

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ACERCA DEL ORGANIZADOR

Ramon González Calvet (1964) es licenciado (1986) y doctor en Química Fundamental por la Universitat de Barcelona (1993). También obtuvo el máster en Matemáticas para profesores por la Universitat Autònoma de Barcelona (1995). Ganó las oposiciones a profesor de matemáticas de secundaria (1987) y fue catedrático de secundaria (2008). Actualmente está jubilado. Ha enseñado álgebra geométrica (de Clifford) a profesores en diversas escuelas de verano, de donde surgió el *Treatise of Plane Geometry through Geometric Algebra* (2007). Durante muchos años hasta el confinamiento, también formó a graduados y profesores interinos que querían ganar las oposiciones a profesor de matemáticas de secundaria. Sus investigaciones se centran principalmente en interfaces electrificadas, álgebra geométrica, el problema de los *n* cuerpos, la geometría diferencial, la mecánica celeste y los relojes de sol. Su aterrizaje en el problema de los *n* cuerpos fue accidental. En su etapa de estudiante en la facultad de química, se dio cuenta de que el hamiltoniano electrónico del átomo de helio no podía ser deducido de ninguna manera lógica, puesto que el problema de los tres cuerpos no tenía solución general conocida. En consecuencia, se planteó y resolvió cómo expresar la energía cinética en términos de las velocidades relativas, lo que le permitió obtener las ecuaciones del movimiento clásico de los tres y *n* cuerpos en términos de coordenadas y aceleraciones relativas. Después, también dedujo los hamiltonianos de los correspondientes problemas cuánticos, que era su objetivo inicial. Aplicó su hamiltoniano de los tres cuerpos al estudio de los niveles de energía vibracional del dióxido de carbono, y de la energía electrónica del átomo de helio, corrigiendo los hamiltonianos dados previamente por otros autores. Después de describir analíticamente el movimiento del sistema Sol-Tierra-Luna en una serie de tres artículos, y de estudiar la dinámica y evolución del sistema solar en una serie de cinco artículos resumidos en el primer capítulo del libro *Planets, Moons, and Beyond: Unveiling the Mysteries of the Solar System* (2026), sus últimos artículos tratan sobre el billar como modelo de la adsorción de moléculas sobre una superficie, y sobre la forma de los glóbulos rojos. Su tesis doctoral sobre termodinámica de interfases electrificadas todavía permanece inédita, aunque su contenido fue parcialmente explicado en algunos artículos.

ÍNDICE ALFABÉTICO

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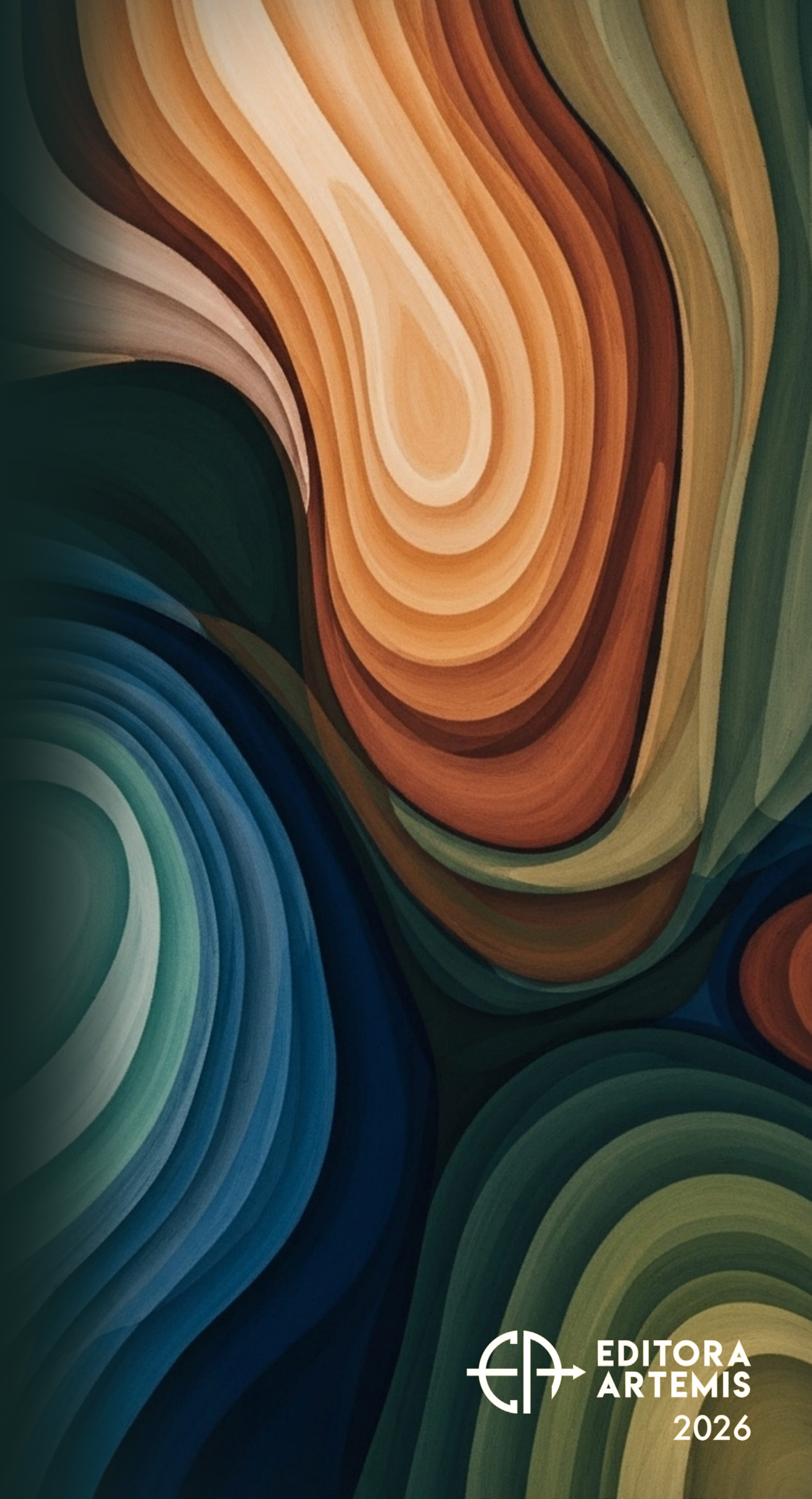

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